Rascal Delta-V Budget

Saint Louis University

Rascal



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Copper Operational

Test Plan

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# Introduction

## Background

The Rascal Delta-V Budget serves to demonstrate the change in velocity (ΔV) necessary for the performance of the Rascal mission. The data collected in this report was calculated through the use of linear orbit theory, as described in Prussing[1] and elaborated upon in Section 1.2, for four different classes of proximity operations and rendezvous:

1. Separation
2. Remote Stationkeeping (RSK)
3. Rendezvous
4. Close-Proximity Stationkeeping, or Inspection

Separation consists of the mechanisms and maneuvers associated with separating the primary spacecraft (the interceptor) from the secondary spacecraft (the target). This involves the change in relative velocity associated with the initial separation sequence (Essentially an instantaneous change in relative spacecraft velocity from zero to some finite value) and the ΔV required to move the interceptor some distance away from the target.

Taxiing consists of maintaining a set relative distance between the interceptor and the target, and thus, the ΔV per orbit required to do so. This type of maneuver, also known as course stationkeeping, is differentiated from fine stationkeeping due to the difference in the relative displacement it is attempting to maintain (100+ meters, as opposed to 10-).

Rendezvous consists of performing a maneuver that both decreases the relative position between the interceptor and the target to some specified value, while simultaneously reducing the relative velocity between each to zero.

Finally, Inspection involves the maintenance of the final relative displacement between the target and the interceptor achieved at the end of the rendezvous maneuver. Like the Taxiing maneuver, this quantity will be measured as a ΔV per orbit.

## Linear Orbit Theory

All ΔV values calculated for the Rascal mission were done so through the use of linear orbit theory. This theory effectively takes the equation of motion for a body exposed to a general gravitational field, where **r** is the inertial position of a spacecraft, is the acceleration of said object spacecraft, **g(r)** is the influence of a general gravitational field, and Γ is the thrust acceleration vector of the spacecraft.

(1)

As opposed to having to analytically solving this second-order, non-linear differential equation (For which several methods and algorithms already exist), one can approximate a solution through the use of linearized equations that describe the motion of one spacecraft relative to each other. This process works well when the relative displacement between a target and interceptor spacecraft is small relative to the overall size of each spacecraft’s orbit (As is the case for the Rascal mission). Though this technique is hindered by the assumption that each spacecraft’s orbit is near-circular, it still offers a useful approximation of the expected ΔV that is to be used in a given mission.

### CW Coordinate Frame and General Linearized Relative Motion

The simplest coordinate frame to utilize for linear orbit theory analysis is that utilized by Clohessy-Wilshire (CW), as shown in Figure 1-1. This coordinate frames is spacecraft-fixed, as opposed to Earth-fixed, meaning that it rotates with the radius vector (**rinertial**) of a given spacecraft. In the case of Rascal, the origin of the coordinate frame is assigned to the target spacecraft. This means that all the relative velocities and positions discussed throughout this overview are defined relative to the target spacecraft’s CW coordinate frame, as shown in Figure 1-2. From this, the relative position between an interceptor and target spacecraft can be defined by the following equation, where **rint** and **rtgt** are the inertial positions of the interceptor and target spacecraft respectively.

CW Coordinate Frame.tif

**Figure 1‑1 Illustration of the Clohessy-Wiltshire coordinate frame.**

(2)

This equation can then be substituted into Equation (1), which, after quite a bit of arithmetic (As fully laid out in Prussing[1]), results in a general solution of the following form:

CW Coordinate Frame Target Spacecraft.tif

**Figure 1‑2 Relative position illustration, with target spacecraft as origin of CW-frame.**

(3)

Where is defined as the 6x1 vector containing both the components of relative spacecraft position (x,y,z) and velocity ( and is the state transition matrix of relative spacecraft motion, which is defined as:

(4)

Where **M(t)**, **N(t)**, **S(t)**, and **T(t)** are each 3x3 matrices defined as:

(5)

With c, s, and n in canonical units being:

(6-8)

With these formulations in mind, one can finally form the general equations for the change in relative position and velocities between a target and interceptor spacecraft with time as:

(9-10)

From here, the total ΔV required to perform a maneuver that defines a new relative displacement between a target and interceptor spacecraft such that, when the interceptor arrives at said new position, the two spacecraft have no net relative velocity, can be defined as:

(11-13)