Rascal Delta-V Budget Overview

Saint Louis University

Rascal



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Copper Operational

Test Plan

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# Introduction

## Background

The Rascal Delta-V Budget serves to demonstrate the change in velocity (ΔV) necessary for the performance of the Rascal mission. The data collected in this report was calculated through the use of linear orbit theory, as described in Prussing[1] and elaborated upon in Section 1.2, for four different classes of proximity operations and rendezvous:

1. Separation
2. Remote Stationkeeping (RSK)
3. Rendezvous
4. Inspection Stationkeeping (ISK)

Separation consists of the mechanisms and maneuvers associated with separating the primary spacecraft (the interceptor) from the secondary spacecraft (the target). This involves the change in relative velocity associated with the initial separation sequence (Essentially an instantaneous change in relative spacecraft velocity from zero to some finite value) and the ΔV required to move the interceptor some distance away from the target.

RSK consists of maintaining a set relative distance between the interceptor and the target, and thus, the ΔV per orbit required to do so. This type of maneuver, also known as course stationkeeping, is differentiated from fine stationkeeping due to the difference in the relative displacement it is attempting to maintain (100+ meters, as opposed to 10-).

Rendezvous consists of performing a maneuver that both decreases the relative position between the interceptor and the target to some specified value, while simultaneously reducing the relative velocity between each to zero.

Finally, ISK involves the maintenance of the final relative displacement between the target and the interceptor achieved at the end of the rendezvous maneuver. Like RSK, this quantity will be measured as a ΔV per orbit.

## Linear Orbit Theory

All ΔV values calculated for the Rascal mission were done so through the use of linear orbit theory. This theory effectively takes the equation of motion for a body exposed to a general gravitational field, where **r** is the inertial position of a spacecraft, is the acceleration of said object spacecraft, **g(r)** is the influence of a general gravitational field, and Γ is the thrust acceleration vector of the spacecraft.

(1)

As opposed to having to analytically solving this second-order, non-linear differential equation (For which several methods and algorithms already exist), one can approximate a solution through the use of linearized equations that describe the motion of one spacecraft relative to each other. This process works well when the relative displacement between a target and interceptor spacecraft is small relative to the overall size of each spacecraft’s orbit (As is the case for the Rascal mission). Though this technique is hindered by the assumption that each spacecraft’s orbit is near-circular, it still offers a useful approximation of the expected ΔV that is to be used in a given mission.

### CW Coordinate Frame and General Linearized Relative Motion

The simplest coordinate frame to utilize for linear orbit theory analysis is that utilized by Clohessy-Wilshire (CW), as shown in Figure 1-1. This coordinate frames is spacecraft-fixed, as opposed to Earth-fixed, meaning that it rotates with the radius vector (**rinertial**) of a given spacecraft. In the case of Rascal, the origin of the coordinate frame is assigned to the target spacecraft. This means that all the relative velocities and positions discussed throughout this overview are defined relative to the target spacecraft’s CW coordinate frame, as shown in Figure 1-2. From this, the relative position between an interceptor and target spacecraft can be defined by the following equation, where **rint** and **rtgt** are the inertial positions of the interceptor and target spacecraft respectively.

CW Coordinate Frame.tif

**Figure 1‑1 Illustration of the Clohessy-Wiltshire coordinate frame.**

(2)

This equation can then be substituted into Equation (1), which, after quite a bit of arithmetic (As fully laid out in Prussing[1]), results in a general solution of the following form:

CW Coordinate Frame Target Spacecraft.tif

**Figure 1‑2 Relative position illustration, with target spacecraft as origin of CW-frame.**

(3)

Where is defined as the 6x1 vector containing both the components of relative spacecraft position (x,y,z) and velocity ( and is the state transition matrix of relative spacecraft motion, which is defined as:

(4)

Where **M(t)**, **N(t)**, **S(t)**, and **T(t)** are each 3x3 matrices defined as:

(5)

With c, s, and n in canonical units being:

(6-8)

With these formulations in mind, one can finally form the general equations for the change in relative position and velocities between a target and interceptor spacecraft with time as:

(9-10)

From here, the total ΔV required to perform a maneuver that defines a new relative displacement between a target and interceptor spacecraft such that, when the interceptor arrives at said new position, the two spacecraft have no net relative velocity, can be defined as:

(11-13)

It is this final result that was used to calculate the total ΔV required to execute each of the maneuvers laid out in Section 1.1.

### Separation ΔV Calculations

For the separation maneuver, it was assumed that, upon separation, the target and interceptor gained an instantaneous relative velocity. This relative velocity was varied between magnitudes of 1 m/s and 20 m/s, as to demonstrate the ΔV associated with various separation cases. It was then assumed that the interceptor spacecraft reached varied final relative displacements between itself and the target, running between 10 and 1000 m.

These conditions were implemented in conjunction with those discussed in Sections 1.2.3-1.2.4 within a single Matlab file (As shown in the Appendix). The results of these calculations for separation are shown in Figure 1-3. As can be seen from the figure, there exist ideal transfer times over which the separation maneuver can take place, mainly, towards the beginning of separation (within the first 5-10 minutes) and towards the end of a single orbit (90-100 minutes). Regardless, a range of ΔV’s for separation can be established between 10-25 m/s.



### Remote and Inspection Stationkeeping ΔV Calculations

Remote stationkeeping is defined as keeping a set relative distance between a target and interceptor spacecraft for a period of orbits. RSK is differentiated from ISK due to the distances involved (100+ meters vs. 10- meters). This means that, on a per orbit basis, some ΔV needs to be expended in order to keep said distance.

The ΔV for this maneuver per orbit was calculated through the use of the final displacement distances

Remote Station Keeping.tif

**Figure 1‑4Total ΔV for remote stationkeeping.** For this case, the stationkeeping distances were set by the final displacement values defined in Figure 1-3 and the initial relative velocity was assumed to be zero.

Separation Between 10 m and 1000 m.tif

**Figure 1‑3Total ΔV required for separation maneuver.** For this case, initial separation velocity magnitudes were varied between 1.00 and 10.0 m/s, while final relative positions were varied between 10 and 1000 m. These values were calculated for transfer times up to 100 minutes.

defined in Section 1.2.2. It was assumed that each stationkeeping maneuver was performed either at the end of the separation maneuver or of another stationkeeping maneuver, implying that the initial relative velocity between the target and the interceptor was zero. A plot of the ΔV associated with these calculations is shown in Figure 1-4. As can be seen from the figure, the optimal transfer time to utilize for a given stationkeeping maneuver is between 90-95 minutes. If one performs a maneuver at this time, the ΔV usage per orbit varies between 0.05-5 m/s.

### Rendezvous ΔV Calculations

Rendezvous is defined as the process of reducing the relative displacement between a target and interceptor to some value for some period of time. This implies having to start the maneuver at some pre-defined relative distance (in this case, that used for RSK), and end at some closer relative distance with a final relative velocity of zero. This is exactly how the total ΔV was calculated for rendezvous maneuvers performed from different distances, the results of which are shown in Figure 1-5. From this graph, its clear that the best time over which to perform the maneuver between 80-85 minutes. For this case, the total ΔV usage varies between 2-15 m/s.

Rendezvous from 10 m to 1000 m.tif

**Figure 1‑5Total ΔV for rendezvous maneuver from initial relative distances between 10 and 1000 m.** In this case, the final relative displacement was set to 10 m, and the initial and final relative velocities were assumed to be 0 m/s.